College of Engineering
Electrical and Computer Department

Communication Systems

(EECE 342: Digital Modulation)

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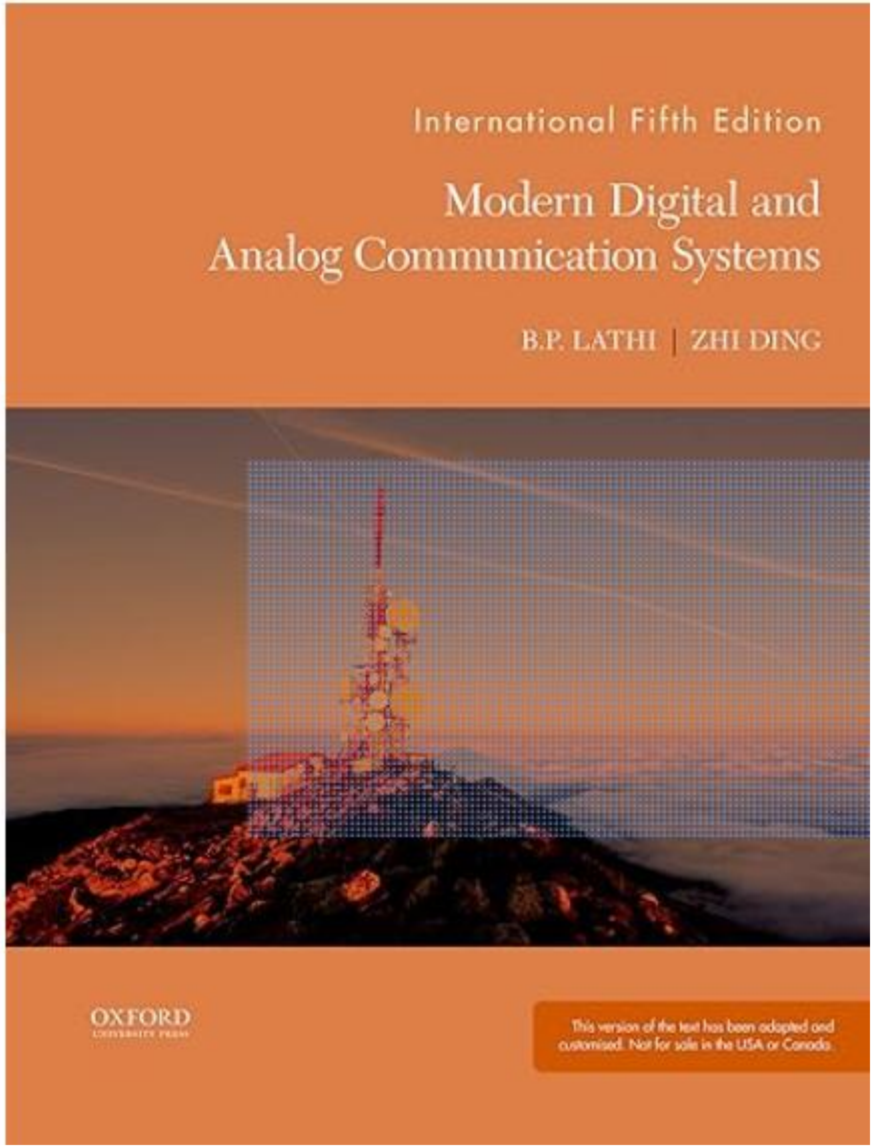
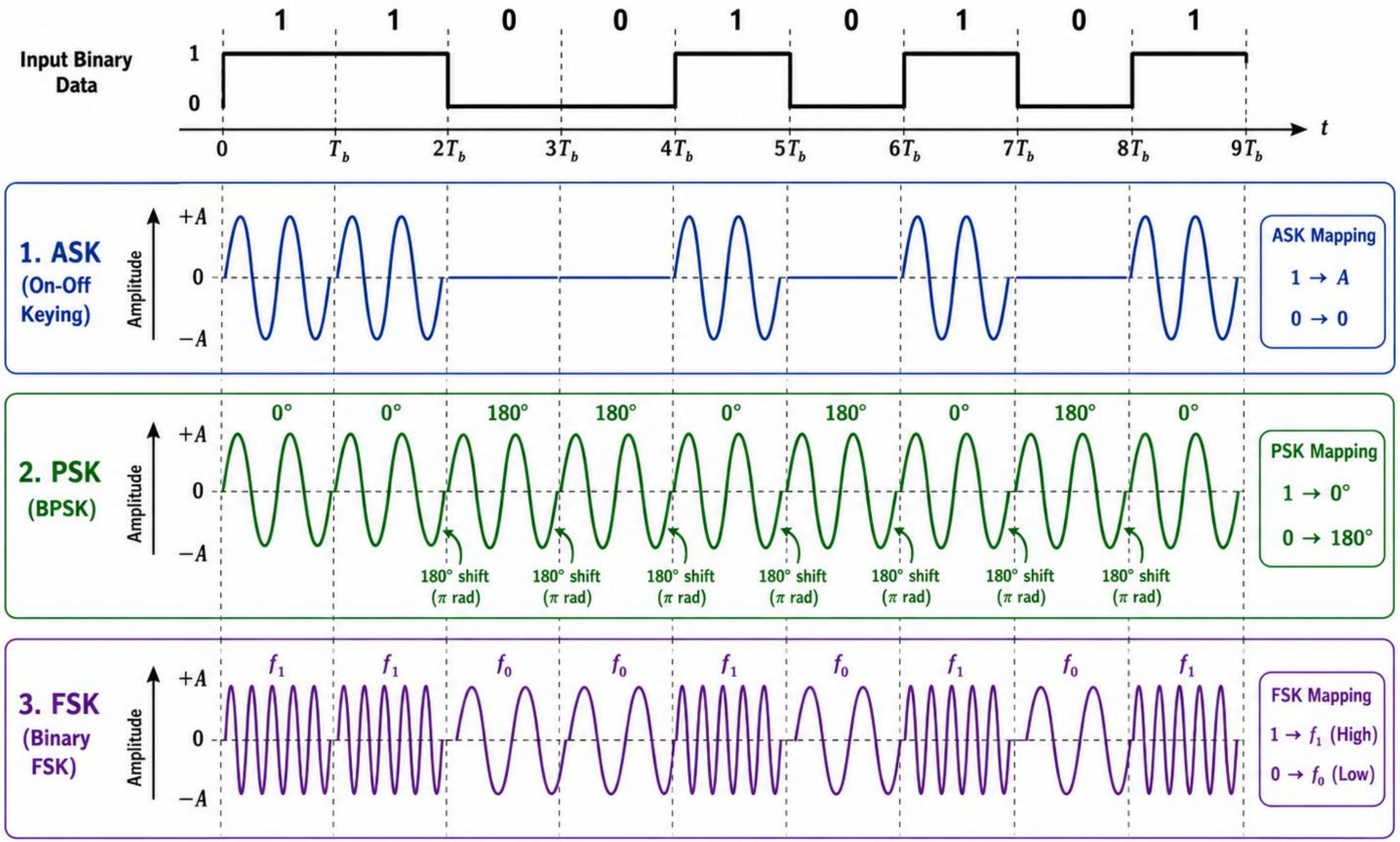
The main outcomes of the course:

- Upon completing this course, the student will be able to

1) Analyze and Design various continuous wave Amplitude modulation and demodulation techniques.
2) Understand the concept of Angle modulation and demodulation and the effect of noise on it.
3) Attain the knowledge about the functioning of different AM, FM Transmitters and Receivers.
4) Analyze and design the various Pulse Modulation Techniques (Analog and Digital Pulse modulation)
5) Understand the concepts of Digital Modulation Technique, Baseband transmission and Optimum Receiver.



Digital Modulation Waveforms for Bit Sequence: 110010101



Key:

T_b = Bit duration (time interval of one bit)

A = Amplitude of the carrier

$f_1 > f_0$ (f_1 = High frequency, f_0 = Low frequency)

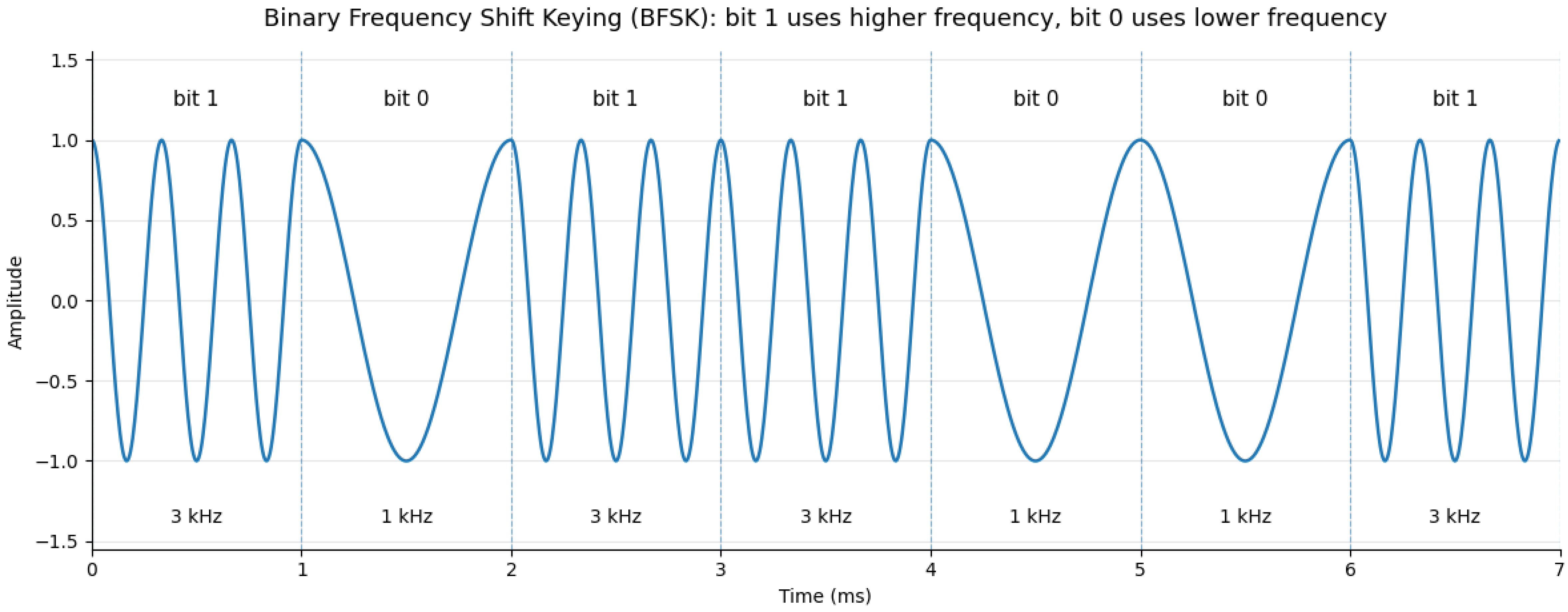
Dashed vertical lines indicate bit boundaries

Summary of Mappings

ASK: $1 \rightarrow A$, $0 \rightarrow 0$

PSK: $1 \rightarrow 0°$, $0 \rightarrow 180°$

FSK: $1 \rightarrow f_1$ (High), $0 \rightarrow f_0$ (Low)


Binary Frequency Shift Keying (BFSK): bit 1 uses higher frequency, bit 0 uses lower frequency
bit 1
bit 0
bit 1
bit 1
bit 0
bit 0
bit 1
3 kHz
1 kHz
3 kHz
3 kHz
1 kHz
1 kHz
3 kHz
Amplitude
Time (ms)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
1
2
3
4
5
6
7

Example: BER of BFSK in AWGN Channel
Assume **orthogonal coherent BFSK** is used.
Given:

$$\frac{E_b}{N_0} = 10\ dB$$

Convert from dB to linear:

$$\frac{E_b}{N_0} = 10^{10/10} = 10$$

For **coherent orthogonal BFSK**, the bit error rate is:

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Substitute:

$$P_b = Q(\sqrt{10})$$
$$P_b = Q(3.162)$$

From the Q-function table:

$$Q(3.162) \approx 7.83 \times 10^{-4}$$

Therefore:

$$\boxed{BER \approx 7.83 \times 10^{-4}}$$

This means approximately:

$$7.83 \times 10^{-4} \times 1{,}000{,}000 = 783$$

So, if **1,000,000 bits** are transmitted:

$$\boxed{\text{Expected } bit\ errors \approx 783\ bits}$$

Interpretation
In BFSK:

Bit *1* → f_1
Bit *0* → f_0

For example:

$$1 \rightarrow 3\ kHz$$
$$0 \rightarrow 1\ kHz$$

So the bit sequence:

$$1011001$$

is transmitted as:

$$f_1,\ f_0,\ f_1,\ f_1,\ f_0,\ f_0,\ f_1$$

The **Q-function** is defined as:

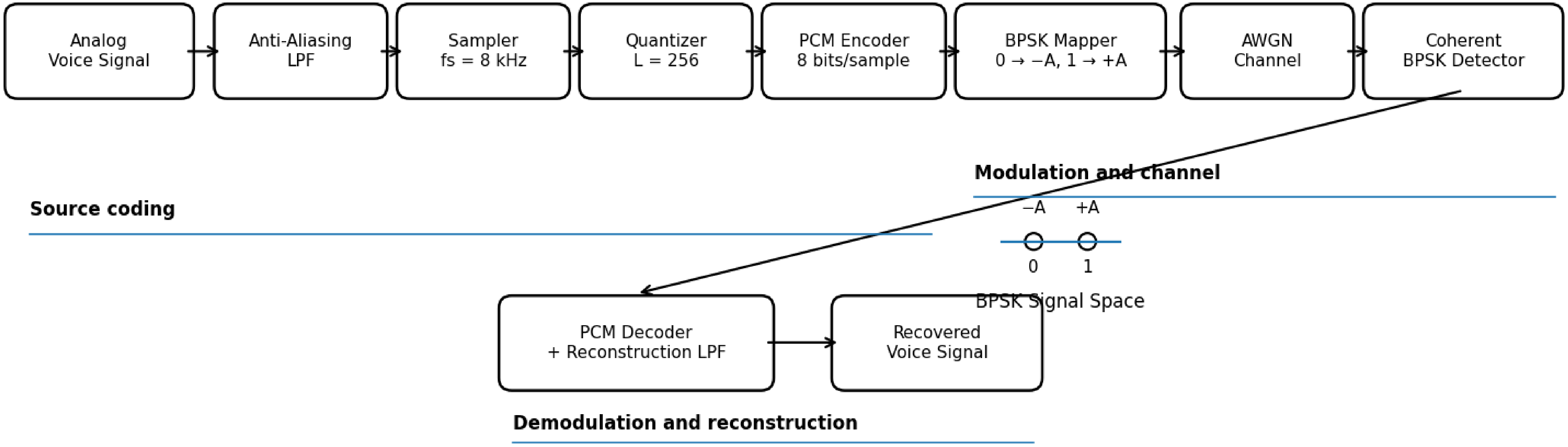
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-u^2/2}\ du$$

It gives the **tail probability** of a standard normal distribution.
Common Q-Function Table

x	$Q(x)$
0.0	0.5000
0.5	0.3085
1.0	0.1587
1.5	0.0668
2.0	0.0228
2.5	0.00621
3.0	0.00135
3.1	0.000967
3.16	0.000782
3.2	0.000687
3.5	0.000233
4.0	0.0000317
4.5	0.00000340
5.0	0.000000287

Digital Voice Transmission Using BPSK

Sampling: 8 kHz | Quantization levels: 256 | PCM rate: 64 kbps | BPSK: 1 bit/symbol



Assume the system uses:

$$f_s = 8\ kHz$$
$$L = 256\ quantization\ levels$$

Because:

$$L = 2^n$$
$$256 = 2^8$$

So each sample needs:

$$n = 8\ bits/sample$$

Step 1: Calculate Bit Rate

$$R_b = f_s \times n$$
$$R_b = 8000 \times 8$$
$$\boxed{R_b = 64000\ bits/s}$$

So the PCM voice signal produces:

$$\boxed{64\ kbps}$$

Since BPSK transmits **one bit per symbol**:

$$\boxed{R_s = R_b = 64\ ksymbols/s}$$

Step 2: Assume Known BER

Assume the known BER of the BPSK system is:

$$BER = 10^{-5}$$

This means:

$$1\ bit\ error\ per\ 100000\ transmitted\ bits$$

Step 3: Expected Number of Errors in One Second

$$N_{bits} = 64000\ bits$$
$$N_{errors} = BER \times N_{bits}$$
$$N_{errors} = 10^{-5} \times 64000$$
$$\boxed{N_{errors} = 0.64\ errors/s}$$

So, on average:

about *1 bit error every 1.56 seconds*

because:

$$\frac{1}{0.64} = 1.56\ s$$

Step 4: Expected Errors in 10 Seconds

$$N_{bits} = 64000 \times 10$$
$$N_{bits} = 640000\ bits$$
$$N_{errors} = 10^{-5} \times 640000$$
$$\boxed{N_{errors} = 6.4}$$

Therefore, during 10 seconds of transmission:

approximately *6 to 7 bit errors*

Assume we use **4-FSK**.

In 4-FSK, there are four possible frequencies:

$$f_0,\ f_1,\ f_2,\ f_3$$

Each frequency represents one symbol.

Because:

$$M = 4$$

$$k = \log_2 M = \log_2 4 = 2$$

So each symbol carries:

$$\boxed{2\ bits/symbol}$$

For example:

Symbol	Frequency	Bits
S_0	f_0	00
S_1	f_1	01
S_2	f_2	10
S_3	f_3	11

Given Data

Assume:

$$N_s = 100000\ transmitted\ symbols$$

Because each symbol carries 2 bits:

$$N_b = 2 \times 100000$$

$$\boxed{N_b = 200000\ transmitted\ bits}$$

Assume the receiver makes:

$$900\ symbol\ errors$$

1. Symbol Error Rate

$$SER = \frac{Number\ of\ wrong\ symbols}{Total\ transmitted\ symbols}$$

$$SER = \frac{900}{100000}$$

$$\boxed{SER = 9 \times 10^{-3}}$$

or:

$$\boxed{SER = 0.009}$$

This means that **9 symbols are wrong out of every 1000 transmitted symbols**.

2. Bit Error Rate

A symbol error may cause **one bit error** or **two bit errors**, because each symbol contains 2 bits.

Assume from the 900 wrong symbols:

Type of Symbol Error	Number of Cases	Bit Errors per Case	Total Bit Errors
One-bit error	600	1	600
Two-bit error	300	2	600

Therefore:

$$\text{Total } bit\ errors = 600 + 600$$

$$\text{Total } bit\ errors = 1200$$

Now:

$$BER = \frac{Number\ of\ wrong\ bits}{Total\ transmitted\ bits}$$

$$BER = \frac{1200}{200000}$$

$$\boxed{BER = 6 \times 10^{-3}}$$

or:

$$\boxed{BER = 0.006}$$

Comparison

Quantity	Value
Modulation	4-FSK
Bits per symbol	2
Transmitted symbols	100000
Transmitted bits	200000
Wrong symbols	900
Wrong bits	1200
Symbol Error Rate	(9 \times 10^{-3})
Bit Error Rate	(6 \times 10^{-3})